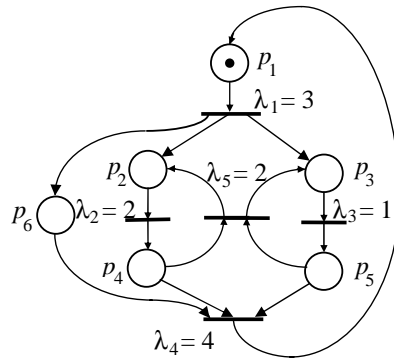


RDCS423 Tutorial Problem and Solutions #4 - Time Augmented and Stochastic Petri nets

- We have a data acquisition system that has a single data channel acquired at a clock rate of $T_1 = 4$ time units. The acquisition time is $T_2 = 3$ time units and the data is pre-processed by different processes that are activated on alternate data points. These different processes perform data conversion ($T_3 = 4$ and $T_5 = 3$ time units respectively) and data normalization ($T_4 = 5$ and $T_6 = 6$ time units respectively). The data is then combined ($T_7 = 1$ time unit) and then a moving average filter process is performed ($T_8 = 2$ time units). Old data used in the filter process is removed by another process ($T_9 = 3$ time units) and the final stage records the processed data point ($T_{10} = 4$ time units).

For this system, produce a Petri net graph to model all processes and augment it with process time information. Derive the constraints imposed on the process times making use of the notion of *safeness in the presence of time*. Determine if the system can achieve the specified time constraints.

- Given the following SPN model for a system with the specified transition rates, determine the average time for token return to place p_1 .



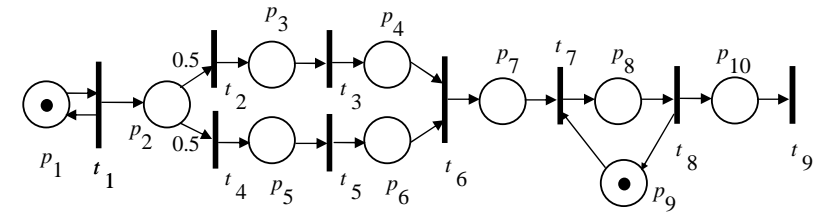
- Given the control software for a two machine flexible manufacturing system with one assembly process each sharing a common tool, and the following transition rates:

- $\lambda_1 = 6$ (process 1 waiting)
- $\lambda_2 = 7$ (process 2 waiting)
- $\lambda_3 = 9$ (process 1 active)
- $\lambda_4 = 10$ (process 2 active)

On average, what fraction of time is the first process using the shared tool as a percentage of the total assembly time?

Solutions:

-



Apply consistency balance to all transitions to find all transition MRFFs F_i :

$$\begin{aligned} \text{Let } F_1 = 1 &\rightarrow F_2 = 0.5, F_4 = 0.5 \\ &\rightarrow F_3 = 0.5, F_5 = 0.5 \rightarrow F_6 = 0.5 \\ &\rightarrow F_7 = 0.5 \rightarrow F_8 = 0.5 \rightarrow F_9 = 0.5 \end{aligned}$$

Then apply the safeness criteria to each net construction:

- p_7 is an entry place to an independent cycle of p_8 and p_9 :
 $T_7 \leq T_1/F_6$ and $T_8 + T_9 \leq T_1/F_6 \Rightarrow T_7 \leq 2T_1$ and $T_8 + T_9 \leq 2T_1$
- p_4 and p_6 are also final places in a parallel synchronous path p_2, p_3, p_4 , and p_2, p_5, p_6 :
 $T_2 + T_3 + T_4 - (T_2 + T_5) \leq T_1/F_5 \Rightarrow T_3 + T_4 - T_5 \leq 2T_1$
 $T_2 + T_5 + T_6 - (T_2 + T_3) \leq T_1/F_3 \Rightarrow T_5 + T_6 - T_3 \leq 2T_1$
- p_2, p_3, p_5, p_8 , and p_{10} are all simple places so that:
 $T_2 \leq T_1/F_1, T_3 \leq T_1/F_2, T_5 \leq T_1/F_4, T_8 \leq T_1/F_7, T_{10} \leq T_1/F_8$

Combining all time constraints and inserting the numerical values values:
 $T_1 = 4, T_2 = 3, T_3 = 4, T_4 = 5, T_5 = 3, T_6 = 6, T_7 = 1, T_8 = 2, T_9 = 3, T_{10} = 4$

$$\begin{aligned} T_4 \leq 2T_1 &\Rightarrow 5 \leq 2(4) \text{ and } T_6 \leq 2T_1 \Rightarrow 6 \leq 2(4) \\ T_8 + T_9 &\leq 2T_1 \Rightarrow 2 + 3 \leq 2(4) \\ T_3 + T_4 - T_5 &\leq 2T_1 \Rightarrow 4 + 5 - 3 \leq 2(4) \\ T_5 + T_6 - T_3 &\leq 2T_1 \Rightarrow 3 + 6 - 4 \leq 2(4) \\ T_2 \leq T_1 &\Rightarrow 3 \leq 4, T_3 \leq 2T_1 \Rightarrow 4 \leq 2(4), T_5 \leq 2T_1 \Rightarrow 3 \leq 2(4) \\ T_7 \leq 2T_1 &\Rightarrow 1 \leq 2(4), T_8 \leq 2T_1 \Rightarrow 2 \leq 2(4), T_{10} \leq 2T_1 \Rightarrow 4 \leq 2(4) \end{aligned}$$

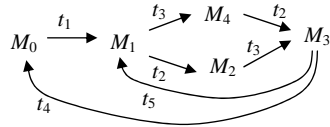
Thus all the time constraints can be met.

2. Find the reachability set:

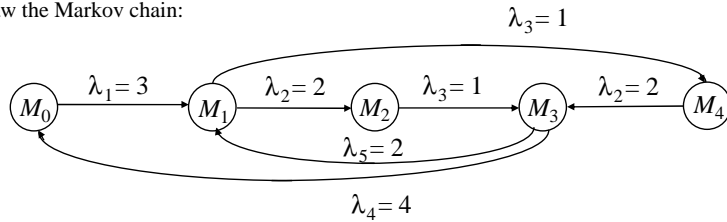
$$M_0 = (1, 0, 0, 0, 0) \quad M_1 = (0, 1, 1, 0, 0, 1) \quad M_2 = (0, 0, 1, 1, 0, 1)$$

$$M_3 = (0, 0, 0, 1, 1, 1) \quad M_4 = (0, 1, 0, 0, 1, 1)$$

And the reachability tree:



Draw the Markov chain:



Applying a 'flow' balance:

$$3P[M_0] = 4P[M_3]$$

$$3P[M_1] = 2P[M_3] + 3P[M_0]$$

$$P[M_2] = 2P[M_1]$$

$$6P[M_3] = P[M_2] + 2P[M_4]$$

$$2P[M_4] = P[M_1]$$

$$P[M_0] + P[M_1] + P[M_2] + P[M_3] + P[M_4] = 1$$

Solve these equations to give:

$$P[M_0] = 0.1429 \quad P[M_1] = 0.2143 \quad P[M_2] = 0.4286$$

$$P[M_3] = 0.1071 \quad P[M_4] = 0.1071$$

and the token occupancy probabilities:

$$P[\mu_1=1] = P[M_0] = 0.1429 \quad P[\mu_2=1] = P[M_1] + P[M_4] = 0.3214$$

$$P[\mu_3=1] = P[M_1] + P[M_2] = 0.6429 \quad P[\mu_4=1] = P[M_2] + P[M_3] = 0.5357$$

$$P[\mu_5=1] = P[M_3] + P[M_4] = 0.2412$$

$$P[\mu_6=1] = P[M_1] + P[M_2] + P[M_3] + P[M_4] = 0.8571$$

As we have token conservation, Little's law can be applied. The utilization of t_1 is $P[\mu_1=1] = 0.1429$ and as $\lambda_1 = 3$ the average token flow in p_1 is 0.4287 tokens/unit time. Due to the fork at t_1 , the flow through the subsystem (composed of p_2, p_3, p_4, p_5, p_6 and t_2, t_3, t_4, t_5) is:

$$\bar{\lambda} = 1.2861.$$

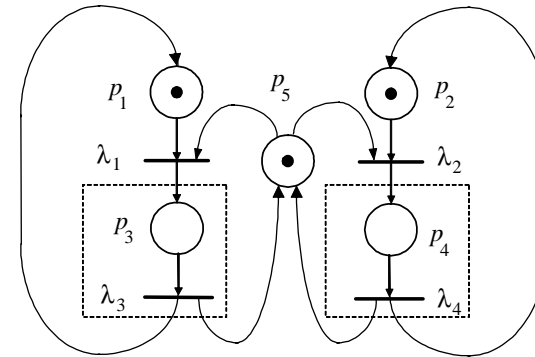
The average number of tokens in the subsystem is given by :

$$\bar{N} = \bar{\mu}_2 + \bar{\mu}_3 + \bar{\mu}_4 + \bar{\mu}_5 + \bar{\mu}_6 = 2.5983$$

The average time a token takes to return to p_1 is given by:

$$\bar{T} = \frac{\bar{N}}{\bar{\lambda}} = 2.02 \text{ time units}$$

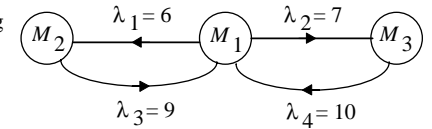
3.



The reachability set is:
 $M_1 = (1, 1, 0, 0, 1)$
 $M_2 = (0, 1, 1, 0, 0)$
 $M_3 = (1, 0, 0, 1, 0)$

and the corresponding Markov chain:

Applying a flow balance to the marking probabilities gives:



$$13P[M_1] = 9P[M_2] + 10P[M_3]$$

$$9P[M_2] = 6P[M_1]$$

$$10P[M_3] = 7P[M_1]$$

$$P[M_1] + P[M_2] + P[M_3] = 1$$

Solve to give:

$$P[M_1] = 0.4225 \quad P[M_2] = 0.2817 \quad P[M_3] = 0.2958$$

and the token occupancy probabilities are:

$$P[\mu_1 = 1] = P[M_1] + P[M_3] = 0.7183$$

$$P[\mu_2 = 1] = P[M_1] + P[M_2] = 0.7042$$

$$P[\mu_3 = 1] = P[M_2] = 0.2817$$

$$P[\mu_4 = 1] = P[M_3] = 0.2958$$

$$P[\mu_5 = 1] = P[M_1] = 0.4225$$

As t_1 requires tokens at p_1 and p_5 to be enabled, the utilization is:

$$P[\mu_1 = 1]P[\mu_5 = 1] = 0.3035$$

As $\lambda_1 = 6$, the average token flow into p_3 (and hence into subsystem 1) is:

$$\bar{\lambda}_1 = 6 \times 0.3035 = 1.8209$$

The average number of tokens in subsystem 1 is $\bar{N}_1 = P[\mu_3 = 1] = 0.2817$

Thus the average time a token spends in p_3 (and hence the average time that the 1st machine assembles a part in process 1) is given by:

$$\bar{T}_1 = \frac{\bar{N}_1}{\bar{\lambda}_1} = 0.1547 \text{ time units}$$

The same approach is used for process 2 to give: $\bar{T}_2 = \frac{\bar{N}_2}{\bar{\lambda}_2} = 0.1420$ time units

Thus process 1 is active $0.1547/(0.1547+0.1420) = 52.1\%$ of the assembly time.