RTDS408 Tutorial Problems & Solutions #1 - Real-Time Scheduling Theory

1. Consider the case of three periodic tasks:

   Task t_1: C_1 = 20 ms; T_1 = 100 ms
   Task t_2: C_2 = 40 ms; T_2 = 150 ms
   Task t_3: C_3 = 100 ms; T_3 = 350 ms

   Apply the Utilization Bound Theorem to determine if these tasks are schedulable using a rate monotonic scheduling strategy. Suppose the computation time for task 1 doubles to 40 msec, now determine if the tasks are schedulable, and then apply the less conservative Completion Time Theorem.

2. Suppose we have four tasks: two periodic, one aperiodic, and one interrupt driven aperiodic. The non-interrupt driven tasks require access to a shared data store, and we wish to give the interrrupt-drive task the highest priority:

   periodic task t_1: C_1 = 30 ms, T_1 = 100 ms
   aperiodic task t_2: C_2 = 30 ms, T_2 = 150 ms
   interrupt driven aperiodic task t_a: C_a = 10 ms, T_a = 200 ms
   periodic task t_3: C_3 = 30 ms, T_3 = 300 ms

   The context switch time is included in the indicated CPU times. Use the Generalized Utilization Bound Theorem to determine if this task set is schedulable.

3. Given two tasks T_1 and T_2 with two shared data structures protected with binary semaphores S_1 and S_2, show how the *priority ceiling protocol* prevents mutual deadlock and guarantees that a high-priority task will be blocked by at most one critical section of any lower priority task.

Solutions:

1. Compute the utilizations for each task:
   Task t_1: C_1 = 20 ms; T_1 = 100 ms $\rightarrow$ U_1 = 0.2
   Task t_2: C_2 = 40 ms; T_2 = 150 ms $\rightarrow$ U_2 = 0.267
   Task t_3: C_3 = 100 ms; T_3 = 350 ms $\rightarrow$ U_3 = 0.286

   i.e. U_{total} = 0.753 Assume that the context switch overhead is included in the CPU times. The upper bound from the Utilization Bound Theorem is:
   $U(3) = 3(2^{1/3}-1) = 0.78$
   which is greater than the total utilization of these tasks $\rightarrow$ all three tasks can meet their deadlines.

   Given that t_1's performance changes to:
   Task t_1: C_1 = 40 ms; T_1 = 100 ms $\rightarrow$ U_1 = 0.4

   Now, U_{total} = 0.953 which is greater than the bound $\rightarrow$ the tasks fail to meet their deadlines. Also the first two tasks can be checked in the same way, e.g. U_{total} = 0.667 and the upper bound becomes:
   $U(2) = 2(2^{1/2}-1) = 0.828$

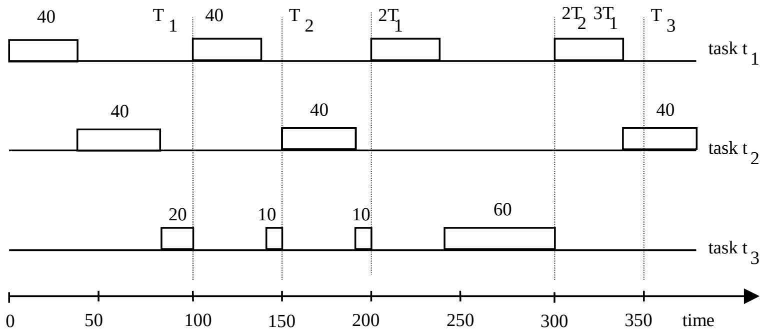
which is greater than the total utilization of these tasks $\rightarrow$ at least the first two tasks can meet their deadlines.

The basis for applying the Completion Time Theorem is that provided each task completes execution before its first period (i.e. meets its first deadline) when all tasks are started at the same time, then the deadlines will be met for any task start times.

The worst-case scenario is with all three tasks ready to execute at the same time. Using rate monotonic scheduling t_1 executes first, followed by t_2 then t_3.

Note that task t_i will execute once for a CPU time of C_i during a period T_i and higher priority tasks will execute more often and may pre-empt task t_i. Thus it is necessary to consider the CPU time used by all higher priority tasks.

The ends of the first periods of each of the tasks, T_i, are referred to as *scheduling points*.



Look at the appropriate scheduling points for task 3, i.e. the ends of the periods of all higher priority tasks which have times less than or equal to task 3's period.

The completion time theorem checks that all tasks have completed their execution at any of the scheduling points (SP). For example, to check if all tasks met their deadlines at task 3's SP, i.e. 350 msec, we potentially have 4 executions of task 1, 3 executions of task 2 and 1 execution of task 3 $\rightarrow$ $4C_1 + 3C_2 + C_3 \le T_3$? (160 + 120 + 100 > 350)

This doesn't mean the task set can't meet its deadlines - the theorem requires that all SP's be checked, i.e. if all tasks can meet their deadlines for one scheduling point, then the task set is schedulable. So looking at each scheduling point:

T_1: $C_1 + C_2 + C_3 \le T_1$? (40 + 40 + 100 > 100)
T_2: $2C_1 + C_2 + C_3 \le T_2$? (80 + 40 + 100 > 150)
$2T_1$: $2C_1 + 2C_2 + C_3 \le 2T_1$? (80 + 80 + 100 > 200)
$3T_1$: $3C_1 + 2C_2 + C_3 \le 3T_1$? (120 + 80 + 100 = 300)
$2T_2$: $3C_1 + 2C_2 + C_3 \le 2T_2$? (120 + 80 + 100 = 300)
T_3: $4C_1 + 3C_2 + C_3 \le T_3$? (160 + 120 + 100 > 350)

Thus the condition for SP $3T_1$ is met, i.e. after 300 msec, task 1 will have run 3 times, task 2 will have run 2 times and task 3 once $\rightarrow$ the required computation fits in this SP.

Note that this indicates that we could not add any higher priority tasks than task 3, otherwise it would miss its deadline. But tasks of lower priority than task 3 could be added if they have a sufficiently long period.

Applying the mathematical expression for the Completion Time Theorem where C_j is the execution time, and T_j is the period, of task t_j:

$$\forall i, 1 \le i \le n, \forall (k,\ p) \in R_i, \quad \sum_{j=1}^{i} C_j \left\lceil \frac{pT_k}{T_j} \right\rceil \quad \le \quad pT_k$$

where $R_i = \{(k,p) \mid 1 \le k \le i,\ p = 1, \cdots, \lfloor T_i / T_k \rfloor\}$ and at least one of the inequalities must be met for each i.

As an example, we must check task t_3 against t_1: i.e. $i = 3$ and $k = 1$

$\rightarrow\ p = 1, \ldots, \lfloor T_i / T_k \rfloor = 1, \ldots, \lfloor 350 / 100 \rfloor = 1, 2, 3.$

i.e. $R_i = (k,p) = (1,1), (1,2), (1,3)$

$$\text{e.g for } (k,\ p) = (1,3) \quad \Rightarrow \quad C_1\left\lceil \frac{(3)T_1}{T_1} \right\rceil + C_2\left\lceil \frac{(3)T_1}{T_2} \right\rceil + C_3\left\lceil \frac{(3)T_1}{T_3} \right\rceil \quad \le \quad (3)T_1$$

$$\Rightarrow C_1\left\lceil \frac{(3)100}{100} \right\rceil + C_2\left\lceil \frac{(3)100}{150} \right\rceil + C_3\left\lceil \frac{(3)100}{350} \right\rceil \quad \le \quad 3T_1$$

$$3C_1 + 2C_2 + C_3 \le 3T_1 \quad \Rightarrow \quad 120 + 80 + 100 = 300$$

and as at least one inequality is met, so the task set is schedulable (strictly, by the theorem, we would also have to check task 2 against task 1, and task 1 against itself).

2. First, determine the utilizations:

periodic task t_1: $C_1 = 30$ ms, $T_1 = 100$ ms $\rightarrow$ $U_1 = 0.3$
aperiodic task t_2: $C_2 = 30$ ms, $T_2 = 150$ ms $\rightarrow$ $U_2 = 0.2$
interrupt driven aperiodic task t_a: $C_a = 10$ ms, $T_a = 200$ ms $\rightarrow$ $U_a = 0.05$
periodic task t_3: $C_3 = 30$ ms, $T_3 = 300$ ms $\rightarrow$ $U_3 = 0.1$

Using rate monotonic priority assignment, the priorities would be t_1, t_2, t_a, t_3. Because a fast response is required to interrupts the priority of t_a is raised to be the highest.

The overall CPU utilization is 0.65 which is less than the utilization bound $U(4) = 4(2^{1/4}-1) = 0.76$. Because of the non-rate monotonic priority assignment it is necessary to consider each task individually:

A. Consider task t_a - highest priority with $U_a = 0.05$ $\rightarrow$ no trouble meeting its deadline.

B. Consider task t_1 - apply the Generalized Utilization Bound Theorem:
a) Pre-emption by high-priority tasks with periods less than T_1 (there are none here).
b) Execution utilization for task t_1 is $U_1 = 0.3$.
c) Pre-emption by high-priority tasks with longer periods. Task t_a falls into this category $\rightarrow$ utilization in the period of the task is $C_a/T_1 = 10$ ms/100 ms = 0.1.

d) Blocking time by lower priority tasks. Both t_2 and t_3 can potentially block t_1 $\rightarrow$ assuming the priority ceiling algorithm is being used, at most only one task can block t_1, so take the worst-case of t_3 (since it has the longer execution time), i.e. blocking utilization during the period of the task is $B_3/T_1 = 30$ ms/100 ms = 0.3.

For task t_1, the worst case utilization = 0.3 + 0.1 + 0.3 = 0.7 which is less than the worst case utilization bound = 0.76 $\rightarrow$ task t_1 will meet it's deadline.

C. Consider task t_2 - apply the Generalized Utilization Bound Theorem:
a) Pre-emption by high-priority tasks with periods less than T_2. Task t_1 has a period less than T_2, so its pre-emption utilization during the period is $U_1 = 0.3$.
b) Execution utilization $U_2 = 0.2$.
c) Pre-emption by high-priority tasks with longer periods. Task t_a falls into this category $\rightarrow$ utilization in the period of the task is $C_a/T_2 = 10$ ms/150 ms = 0.07.
d) Blocking time by lower priority tasks. Task t_3 can potentially block t_2 $\rightarrow$ again assuming the priority ceiling algorithm is being used, at most only one task can block t_2, so take the worst-case of t_3, i.e. blocking utilization during the period of the task is $B_3/T_2 = 30$ ms/150 ms = 0.2.

For task t_2, the worst case utilization = 0.3 + 0.2 + 0.07 + 0.2 = 0.77 which is greater than the worst case utilization bound = 0.76 $\rightarrow$ task t_2 will just miss its deadline.

D. Consider task t_3 - apply the Generalized Utilization Bound Theorem:
e) Pre-emption by high-priority tasks with periods less than T_3. Tasks t_1, t_2 and t_a have periods less than T_3, so its pre-emption utilization during the period is $U_1 + U_2 + U_a = 0.3 + 0.2 + 0.05 = 0.55$.
f) Execution utilization $U_3 = 0.1$.
g) Pre-emption by high-priority tasks with longer periods. There are no tasks in this category.
h) Blocking time by lower priority tasks. There are no lower priority tasks.

For task t_3, the worst case utilization = 0.55 + 0.1 = 0.65 which is less than the worst case utilization bound = 0.76 $\rightarrow$ task t_3 will meet its deadline.

Thus all four tasks will not meet their deadlines (although task 2 is close to meeting its deadline). As usually the worst case upper bound is taken to be 0.69 to provide a reasonable safety-margin $\rightarrow$ requires rescheduling of tasks.

3. Let T_1 attempt to lock the semaphores in the order S_1 then S_2 and T_2 attempt to lock in the reverse order. Also let T_1 have higher priority than T_2. As both tasks use the semaphores, the priority ceilings of S_1 and S_2 must be set to that of task T_1 or higher.

Let T_2 start by acquiring S_2 and when T_1 is executed it will pre-empt T_2. T_1 attempts to lock S_1 but because its priority is not higher than the priority ceiling of already locked semaphore S_2 it is suspended. Task T_2 has its priority raised to that of the semaphore it has acquired (S_2) and it now continues execution and acquires S_1.

Only one lower priority task can block T_1 because when the semaphore that the lower priority task has (in this case T_2) is returned, T_1 continues execution with that semaphore (since it was suspended at that point and is the highest priority task waiting on that semaphore).